Spring 2020

SABANCI UNIVERSITY

# Faculty of Engineering and Natural Sciences

# CS 302 Automata Theory

***ANSWERS***

***Answer 1*** (*25* points)

***(a)*** *We approach the problem by designing an NFA that accepts* ***Lc (****the complement of* ***L)*** *which is given by the following regular expression* ***E***

***E*** *= (1+0)\*.0.0.0.(1+0)\**

*This is accepted by the NFA* ***C*** *below*

*0,1*

***C >***

*0,1*

0

*0*

*0*

***1***

***2***

***3***

***4***

*The DFA corresponding to* ***C*** *above is as below*

|  |  |  |
| --- | --- | --- |
| ***State*** | ***input = 0*** | ***input = 1*** |
| ***> 1=X*** | ***1,2 =Y*** | ***1*** |
| ***1,2*** | ***1,2,3 =Z*** | ***1*** |
| ***1,2,3*** | ***1,2,3,4 =U\**** | ***1*** |
| ***1,2,3,4*** | ***1,2,3,4*** | ***1, 4 =V\**** |
| ***1, 4*** | ***1,2,4=W\**** | ***1,4*** |
| ***1,2,4*** | ***1,2,3,4*** | ***1,4*** |

***A*** *is obtained from* ***C*** *by interchanging the final and non-final states to account for the complement language as shown below.*

***A*** >

*1*

*1*

***V***

*0*

*0*

*0*

*1*

*1*

*0*

*1*

*0*

***X***

***Y***

***Z***

***U***

*0*

***W***

*1*

***(b)*** *The minimal state machine* ***B*** *is obtained using the table filling algorithm as below*

*where* ***U ≡ V ≡ W***.

*X Y Z U V W*

***B* >**

*1*

*0,1*

*0*

*1*

*0*

*1*

*0*

***X***

***Y***

***Z***

***U,V,W***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | *3* | *2* | *1* | *1* | *1* |
|  |  | *2* | *1* | *1* | *1* |
|  |  |  | *1* | *1* | *1* |
|  |  |  |  | *-* | *-* |
|  |  |  |  |  | *-* |
|  |  |  |  |  |  |

***Answer 2*** (*25* points)

***(a)*** *(10 pts)* The language corresponding to income exceeding ***0*** TL is :

***L = (w∈{0,1}\* | #1****’s*in ***w > #0****’s*in ***w )***

Since this is not a regular language an *NFA* cannot be used for detection. Proof of this is straightforward using the *pumping lemma* for regular languages.

***(b)*** *(15 pts) A PDA* ***P = ({q0 , f} ,{0,1} ,{0,1,Z0}, δ , Z0 , q0 , {f} )*** *where* ***δ*** *is given by :*

***(q0 , 0 , Z0 ) 🡪 (q0 ,0Z0)***

***(q0 , 0 ,0) 🡪 (q0 , 00)***

***(q0 ,1,0) 🡪 (q0 ,e)***

***(q0 ,1, Z0) 🡪 (f , Z0 )***

***(f ,0, Z0) 🡪 (q0 , Z0)***

***(f ,1, Z0) 🡪 (f , 1Z0)***

***(f ,1, 1) 🡪 (f , 11)***

***(f ,0, 1) 🡪 (f , e)***

*Then* ***P*** *accepts (or detects) the winning sequence by* ***final state****. Moreover* ***P*** *is a DPDA.*

*(Note that according to the phrasing of the question the ‘bell’ rings after a transition* ***iff P*** *is in the final state* ***f****.)*

***Answer 3*** (*25 points)*

*A CFG* ***G = ({E,T,F,I} , {+,\*, ( , ) , x,y,z} , R, E)*** *where the productions* ***R*** *are as follows :*

***E 🡪 E + T | T ; T 🡪 T\*F | F ; F 🡪 I |(E) | I(E) ; I 🡪 x|y|z***

***(a)*** *There are no nullable variables hence we start with unit pairs:*

***(E,T),(E,F),(E,I),(T,F) ,(T,I) ,(F,I)*** *hence we have the following productions :*

***E 🡪 E + T | T\*F | (E)| I(E) | x|y|z***

***T 🡪 T\*F | ( E ) | I( E ) |x|y|z***

***F 🡪 ( E ) | I( E ) | x|y|z***

*Next step is to replace non-single RHS terminals by non-terminals*

*Plus 🡪 +*

*Mult 🡪 \**

*[ 🡪 (*

*] 🡪 )*

*Hence*

***E 🡪 E Plus T | T Mult F | [E] | I[E] | x|y|z***

***T 🡪 T Mult F | [ E ] | I [ E ] |x|y|z***

***F 🡪 [E ] | I [ E ] | x|y|z***

*After ensuring two items on RHS the final CNF is given below*

***E 🡪 E A |T B | [ C | I D | x|y|z ; A 🡪 Plus T ; B 🡪 Mult F ; C 🡪 E ] ; D 🡪 [C***

***T 🡪 TB | [ C | I D | x|y|z***

***F 🡪 [ C | I D | x|y|z;***

***I 🡪 x|y|z***

*Plus 🡪 +*

*Mult 🡪 \**

*[ 🡪 (*

*] 🡪 )*

***(b)*** *(13 pts)*

*Given :*

***E 🡪 E + T | T ; T 🡪 T\*F | F ; F 🡪 I |(E) | I(E) ; I 🡪 x|y|z***

*First remove left recursions by right recursions*

***E 🡪 E + T | T*** *by :* ***E 🡪 T B ; B 🡪 +T B | e***

***T 🡪 T\*F | F*** *by* ***: T 🡪 F C ; C 🡪 \* F C | e***

*Then substitute by using appropriate productions eventually to replace the first nonterminals in a production by terminals.*

***E 🡪 FCB*** *or* ***E 🡪 ICB |(E)CB | I(E)CB*** *or:*

***E 🡪 x-y-z-CB | (E)CB | x-y-z (E) CB*** *. . . (1)*

***T*** *🡪* ***FC*** *or*

***T 🡪 x-y-z-C | (E)C | x-y-z (E) C*** *. . . (2)*

***F*** *🡪* ***x-y-z | (E) | x-y-z (E)*** *. . . (3)*

*Finally get rid of the nullable productions for* ***B*** *and* ***C***

***B 🡪 +T B | +T*** *. . . (4)*

***C 🡪 \* F C | \*F*** *. . . (5)*

*and in (1) and (2) replace* ***CB*** *(and* ***C****) by 4 possibilities (2 possibilities) where none; one or both of* ***B*** *and* ***C*** *are replaced by* ***e****.*

*(1),(2), (3), (4) and (5) constitute the GNF*

***Answer 4 (****25* points)

***(a)*** *(10 pts)*

|  |  |  |
| --- | --- | --- |
| ***Label TM*** | ***Condition*** | ***TM*** |
| ***A > R*** | ***σ = 0*** | ***1.A*** |
|  | ***σ = 1*** | ***0.A*** |
|  | ***σ = #*** | ***L# .h*** |

***(b)*** *(15 pts)*

|  |  |  |
| --- | --- | --- |
| ***Label TM*** | ***Condition*** | ***TM*** |
| ***A = R1#*** | ***-*** | ***B*** |
| ***B=L1.R2*** | ***σ1 = x ≠ #*** | ***x2.B*** |
|  | ***σ = #*** | ***L2# .C*** |
| ***C = R1 .R2*** | ***σ1 = σ2 ≠ #*** | ***C*** |
|  | ***σ1 = σ2 = #*** | ***hYES*** |
|  | ***else*** | ***hNO*** |